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Tachyon kink on non-BPS Dp-brane in the general background

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Abstract: This paper is devoted to the study of the tachyon kink on the worldvolume of a non-BPS Dp-brane that is embedded in general background, including $NS-NS$ two form B and also general Ramond-Ramond field. We will explicitly show that the dynamics of the kink is described by the equations of motion that arrise from the DBI and WZ action for D(p-1)-brane.

Keywords: Tachyon Condensation, D-branes.

Contents

1. Introduction and summary

The study of the open string tachyon brought significant progress in the understanding of the nonperturbative aspect of string theory.[1]. Among many results that were obtained in the past there is a very interesting observation that shows that some aspects of the tachyon condensation can be correctly captured by the effective field theory description, where the tachyon effective action (2.1), describing the dynamics of the tachyon field on a non-BPS Dp-brane of type-IIA and IIB theory was proposed in [6 – 9].[2]

One of the well known solutions of the tachyon effective field theory is a kink solution which is supposed to describe a BPS D(p-1)-brane [15 – 22]. Very nice analysis of the kink solution was performed in the paper [15] where it was shown that the energy density of the kink in the effective field theory is localised on codimension one surface as in the case of a BPS D(p-1)-brane. It was then also shown that the worldvolume theory of the kink solution is also given by the Dirac-Born-Infeld (DBI) action on a BPS D(p-1)-brane. This result demonstrates that the tachyon effective action reproduces the low energy effective action on the world-volume of the soliton.

In our recent paper [30] we have extended this analysis to the spatial dependent tachyon condensation on a unstable Dp-brane moving in nontrivial background with the diagonal form of the metric.[3] We have shown that this form of the tachyon condensation leads to an emergence of a D(p-1)-brane where the scalar modes that propagate on the kink worldvolume are solutions of the equations of motion that arise from the DBI action for D(p-

[1] For review of the open string tachyon condensation, see [1 – 5].

[2] For recent discussion of the effective field theory description of the tachyon condensation, see [10 – 14].

[3] The similar problem was previously studied in [16, 23, 24].

1)-brane that is moving in given background and that is localised at the core of the kink.

The purpose of this paper is to extend this analysis to the general background, including $NS-NS$ two form field and Ramond-Ramond forms as well. We will study this problem in two ways. In the first one we will consider a non-BPS Dp-brane action where the worldvolume diffeomorphism is not fixed at all. The analysis of the equation of motion in this way is straightforward and in some sense demonstrates the efficiency of the study of the Dp-brane dynamics without imposing any gauge fixing conditions.[4] More precisely, we will show that the spatial dependent tachyon condensation leads to an emergence of a D(p-1)-brane whose dynamics is governed by equation of motion that arise from the DBI and WZ action for D(p-1)-brane. We will also show that the mode that characterises the core of the kink does not depend on the worldvolume coordinates of the kink and that all its values are equivalent. This result is consistent with the fact that we do not presume any relations between worldvolume coordinates and target space ones so that all positions of the kink on the worldvolume of a unstable Dp-brane are equivalent.

In the second approach we use the diffeomorphism invariance so that we will presume that the worldvolume coordinate that parametrises the spatial dependent tachyon condensation is equal to one spatial coordinate in target spacetime. We will then demonstrate that the dynamics of the kink solution is governed by the equation of motion of D(p-1)-brane even if the analysis of these equations is more difficult. We will also show that the mode that describes location of the kink on the worldvolume of a non-BPS Dp-brane has physical meaning as the embedding coordinate in the spatial direction that coincides with the worldvolume direction. We will also show that this mode obeys the equation of motion that arises from the DBI and WZ term for D(p-1)-brane moving in given background.

These results explicitly demonstrate that the tachyon like DBI action together with WZ term allows correct description of the emergence of a BPS D(p-1)-brane. We also hope that this analysis could be extended to other situations where the effective field theory description of the tachyon condensation could be useful. For example, we would like to apply this analysis to the supersymmetric version of a non-BPS Dp-brane in general background, following again [15]. It would be also nice to find solution of the tachyon equation of motion that describes D-branes with codimensions larger then one. In other words, we would like to see whether we can describe an emergence of a D(p-2)-brane that, by definition is unstable and hence the tachyon should be present on the worldvolume of the kink.

The rest of this paper is organised as follows. In the next section 2 we will analyse the equation of motion for non -BPS Dp-brane in curved background without any static gauge presumption. We will see that the modes living on the worldvolume of the kink solve the equation of motion that arise from the DBI and WZ action for BPS D(p-1)-brane. We will also calculate the stress energy tensor and we will show that it is equal to the stress energy tensor for D(p-1)-brane. In section 3 we will study the same problem where now we partially fix the gauge. We will again show that the dynamics of the kink is governed by the DBI and WZ action for BPS D(p-1)-brane.

[4]We thank prof. U. Lindström for stressing this point to us.

2. Non-BPS Dp-brane in general background

As in our previous paper we begin with the Dirac-Born-Infeld like tachyon effective action in general background [6 − 9]

$$
\begin{aligned}
S &= -\int d^{p+1}\xi e^{-\Phi} V(T)\sqrt{-\det \mathbf{A}}\,, \\
\mathbf{A}_{\mu\nu} &= g_{MN}\partial_\mu X^M \partial_\nu X^N + b_{MN}\partial_\mu X^M \partial_\nu X^N + F_{\mu\nu} + \partial_\mu T \partial_\nu T\,, \qquad \mu,\nu = 0,\dots,p\,, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu\,,
\end{aligned} \tag{2.1}
$$

where $A_\mu, \mu, \nu = 0, \dots, p$ and $X^{M,N}, M, N = 0, \dots, 9$ are gauge and the transverse scalar fields on the worldvolume of the non-BPS Dp-brane and T is the tachyon field. $V(T)$ is the tachyon potential that is symmetric under $T \to -T$ has maximum at $T = 0$ equal to the tension of a non-BPS Dp-brane τ_p and has its minimum at $T = \pm\infty$ where it vanishes.

Since we will consider a non-BPS Dp-brane in the background with nontrivial Ramond-Ramond field we should also include the Wess-Zumino (WZ) term for non-BPS Dp-brane that is supposed to have a form [25][5]

$$
S_{WZ} = \int_\Sigma V(T) dT \wedge C e^{F+B}\,. \tag{2.2}
$$

In (2.2) Σ denotes the worldvolume of a non-BPS Dp-brane and C collects all RR n-form gauge potentials (pulled back to the worldvolume) as

$$
C = \oplus_n C_{(n)}\,. \tag{2.3}
$$

The form of the WZ term (2.2) was determined from the requirement that the Ramond-Ramond charge of the tachyon kink is equal to the charge of D(p-1)-brane.[6]

Using (2.1) and (2.2) we now obtain the equations of motion for T, X^M and A_μ. The equation of motion for tachyon takes the form

$$
-e^{-\Phi}V'(T)\sqrt{-\det \mathbf{A}} + \frac{1}{2}\partial_\mu\left[e^{-\Phi}\partial_\nu T\left((\mathbf{A}^{-1})^{\nu\mu} + (\mathbf{A}^{-1})^{\mu\nu}\right)\sqrt{-\det \mathbf{A}}\right] + J_T = 0\,, \tag{2.4}
$$

where $J_T = \frac{\delta}{\delta T}S_{WZ}$ is the source current derived from varying the Wess-Zumino term. For scalar modes we obtain

$$
\begin{aligned}
&-\frac{\delta e^{-\Phi}}{\delta X^K} V\sqrt{-\det \mathbf{A}} - \\
&\quad -\frac{e^{-\Phi}}{2} V\left(\frac{\delta g_{MN}}{\delta X^K}\partial_\mu X^M \partial_\nu X^N + \frac{\delta b_{MN}}{\delta X^K}\partial_\mu X^M \partial_\nu X^N\right)(\mathbf{A}^{-1})^{\nu\mu}\sqrt{-\det \mathbf{A}} + \\
&\quad + \frac{1}{2}\partial_\mu\left[e^{-\Phi} V g_{KM}\partial_\nu X^M\left((\mathbf{A}^{-1})^{\nu\mu} + (\mathbf{A}^{-1})^{\mu\nu}\right)\sqrt{-\det \mathbf{A}}\right] + \\
&\quad + \frac{1}{2}\partial_\mu\left[e^{-\Phi} V b_{KM}\partial_\nu X^M\left((\mathbf{A}^{-1})^{\nu\mu} - (\mathbf{A}^{-1})^{\mu\nu}\right)\sqrt{-\det \mathbf{A}}\right] + J_K = 0\,,
\end{aligned} \tag{2.5}
$$

[5] We work in units $2\pi\alpha' = 1$.

[6] Another aspects of Wess-Zumino term for non-BPS Dp-brane were also discussed in [26 − 29].

where $J_K = \frac{\delta}{\delta X^K} S_{WZ}$. Finally, the equations of motion for A_μ are

$$\frac{1}{2}\partial_\nu \left[e^{-\Phi} V \left((\mathbf{A}^{-1})^{\mu\nu} - (\mathbf{A}^{-1})^{\nu\mu} \right) \sqrt{-\det \mathbf{A}} \right] + J^\mu = 0 \,, \tag{2.6}$$

where $J_\mu = \frac{\delta}{\delta A_\mu} S_{WZ}$. To simplify notation it is convenient to introduce the symmetric and antisymmetric form of the matrix $(\mathbf{A}^{-1})$

$$(\mathbf{A}^{-1})^{\nu\mu}_S = \frac{1}{2}((\mathbf{A}^{-1})^{\nu\mu} + (\mathbf{A}^{-1})^{\mu\nu}) \,, \qquad (\mathbf{A}^{-1})^{\nu\mu}_A = \frac{1}{2}((\mathbf{A}^{-1})^{\nu\mu} - (\mathbf{A}^{-1})^{\mu\nu}) \,. \tag{2.7}$$

Now we derive the explicit form of the currents that arise from the WZ term (2.2). To do this we write (2.2) as (We will closely follow the analysis of the currents for BPS Dp-brane that was performed in [31].)

$$S_{WZ} = \sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \int d^{p+1}\xi \epsilon^{\mu_1 \cdots \mu_{p+1}} V(T) \left((\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T \right) , \tag{2.8}$$

where $\epsilon^{\mu_1 \cdots \mu_{p+1}}$ is Levi-Civita tensor (with no metric factors) and $q = (2p + 1 - 1 - 2n)$. The explicit variation of (2.8) is equal to

$$\begin{aligned}
\delta S_{WZ} = &\sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \times \\
&\times \int d^{p+1}\xi \epsilon^{\mu_1 \cdots \mu_{p+1}} \Big[V'(T) \delta T \left((\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T \right) + \\
&\qquad + V(T) \left((\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} \delta T \right) + \\
&\qquad + V(T) \big(n (2 \partial_{\mu_1} \delta A_{\mu_2} + \partial_K b_{MN} \delta X^K \partial_{\mu_1} X^M \partial_{\mu_2} X^N + \\
&\qquad\qquad + 2 b_{MN} \partial_{\mu_1} \delta X^M \partial_{\mu_2} X^N) (\mathcal{F})^{n-1}_{\mu_3 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T \big) + \\
&\qquad + V(T) (\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} (q C_{M_1 \cdots M_q} \partial_{\mu_{2n+1}} \delta X^{M_1} \cdots \partial_{\mu_p} X^{M_q} + \\
&\qquad\qquad\qquad + \partial_M C_{M_1 \cdots M_q} \times \\
&\qquad\qquad\qquad \times \delta X^M \partial_{\mu_{2n+1}} X^{M_1} \cdots \partial_{\mu_p} X^{M_q}) \partial_{\mu_{p+1}} T \Big] \,.
\end{aligned} \tag{2.9}$$

From this equation we obtain following form of the currents

$$J^{\mu_1} = \sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{\mu_1 \cdots \mu_{p+1}} \partial_{\mu_2} \left[V(T) (\mathcal{F})^{n-1}_{\mu_3 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T \right] , \tag{2.10}$$

$$\begin{aligned}
J_T = &\sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \epsilon^{\mu_1 \cdots \mu_{p+1}} V'(T) \left((\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T \right) - \\
&- \partial_{\mu_{p+1}} \sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \epsilon^{\mu_1 \cdots \mu_{p+1}} \left[V(T) (\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \right] .
\end{aligned} \tag{2.11}$$

and

$$\begin{aligned}
J_K = &\sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \epsilon^{\mu_1 \cdots \mu_{p+1}} \times \\
&\times \big[V(T) \partial_K b_{MN} \partial_{\mu_1} X^M \partial_{\mu_2} X^N (\mathcal{F})^{n-1}_{\mu_3 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T + \\
&\quad + V(T) (\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} \partial_K C_{M_1 \cdots M_q} \partial_{\mu_{2n+1}} X^{M_1} \cdots \partial_{\mu_p} X^{M_q} \partial_{\mu_{p+1}} T - \\
&\quad - 2n \partial_{\mu_1} \left[V(T) b_{KM} \partial_{\mu_2} X^M (\mathcal{F})^{n-1}_{\mu_3 \cdots \mu_{2n}} C_{\mu_{2n+1} \cdots \mu_p} \partial_{\mu_{p+1}} T \right] - \\
&\quad - q \partial_{\mu_{2n+1}} \left[V(T) (\mathcal{F})^n_{\mu_1 \cdots \mu_{2n}} C_{K M_2 \cdots M_q} \partial_{\mu_{2n+2}} X^{M_2} \cdots \partial_{\mu_p} X^{M_q} \partial_{\mu_{p+1}} T \right] \big] \,.
\end{aligned} \tag{2.12}$$

Now we try to find the solution of the equations of motion (2.4), (2.5) and (2.6) that can be interpreted as a lower dimensional D(p-1)-brane moving in given background. Without lost of generality we choose one particular worldvolume coordinate, say $\xi^p \equiv x$ and consider following ansatz for the tachyon

$$T(x,\xi) = f(a(x - t(\xi)) , \tag{2.13}$$

where as in [15] we presume that $f(u)$ satisfies following properties

$$f(-u) = -f(u) , \quad f'(u) > 0 , \quad \forall u , \qquad f(\pm\infty) = \pm\infty \tag{2.14}$$

but is otherwise an arbitrary function of its argument u. a is a constant that we shall take to ∞ in the end. In this limit we have $T = \infty$ for $x > t(\xi)$ and $T = -\infty$ for $x < t(\xi)$. Note also that $t(\xi)$ in (2.13) is function of $\xi^\alpha, \alpha = 0, \dots, p-1$. Let us also presume following ansatz for massless fields

$$X^M(x,\xi) = X^M(\xi) , \qquad A_x(x,\xi) = 0 , \qquad A_\alpha(x,\xi) = A_\alpha(\xi) , \qquad \alpha = 0, \dots, p-1 , \tag{2.15}$$

where again $\xi \equiv (\xi^0, \dots, \xi^{p-1})$. Before we proceed further we would like to stress what is the main goal of this analysis. We would like to show that the dynamics of the kink is governed by the action

$$\begin{aligned} S &= S_{DBI} + S_{WZ} , \\ S_{DBI} &= -T_{p-1} \int d^p\xi e^{-\Phi} \sqrt{-\det \tilde{\mathbf{a}}} , \\ S_{WZ} &= \sum_{n \geq 0} \frac{1}{n!(2!)^n q!} \int d^p\xi \epsilon^{\alpha_1 \cdots \alpha_p} (\tilde{\mathcal{F}})^n_{\alpha_1 \cdots \alpha_{2n}} \tilde{C}_{\alpha_{2n+1} \cdots \alpha_p} , \end{aligned} \tag{2.16}$$

where

$$\begin{aligned} \tilde{\mathbf{a}}_{\alpha\beta} &= (g_{MN} + b_{MN}) \partial_\alpha X^M \partial_\beta X^N + F_{\alpha\beta} , \\ \tilde{\mathcal{F}}_{\alpha\beta} &= F_{\alpha\beta} + b_{MN} \partial_\alpha X^M \partial_\beta X^N , \\ \tilde{C}_{\alpha_{2n+1} \cdots \alpha_p} &= C_{M_{2n+1} \cdots M_p} \partial_{\alpha_{2n+1}} X^{M_{2n+1}} \cdots \partial_{\alpha_p} X^{M_p} . \end{aligned} \tag{2.17}$$

In other words we will show that the modes given in (2.15) that propagate on the world-volume of the kink obey the equations of motion derived from (2.16) that have the form

$$\begin{aligned} -\frac{\delta e^{-\Phi}}{\delta X^K} V \sqrt{-\det \mathbf{A}} - \frac{e^{-\Phi}}{2} \left(\frac{\delta g_{MN}}{\delta X^K} \partial_\alpha X^M \partial_\beta X^N + \frac{\delta b_{MN}}{\delta X^K} \partial_\alpha X^M \partial_\beta X^N \right) (\tilde{\mathbf{a}}^{-1})^{\beta\alpha} \sqrt{-\det \tilde{\mathbf{a}}} + \\ + \partial_\alpha \left[e^{-\Phi} g_{KM} \partial_\beta X^M (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S \sqrt{-\det \tilde{\mathbf{a}}} \right] + \\ + \partial_\alpha \left[e^{-\Phi} b_{KM} \partial_\beta X^M (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A \sqrt{-\det \tilde{\mathbf{a}}} \right] + \tilde{J}_K = 0 , \end{aligned} \tag{2.18}$$

where

$$\tilde{J}_K = \frac{\delta S_{WZ}}{\delta X^K}$$

$$
\begin{aligned}
&= \sum_{n\geq 0} \frac{1}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p} \times \\
&\times \Big[\partial_K b_{MN}\partial_{\alpha_1}X^M\partial_{\alpha_2}X^N(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}+ \\
&\quad +(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\partial_K\tilde{C}_{M_1\cdots M_q}\partial_{\alpha_{2n+1}}X^{M_1}\cdots\partial_{\alpha_p}X^{M_q}- \\
&\quad -2n\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}X^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]- \\
&\quad -q\partial_{\alpha_{2n+1}}\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}C_{KM_2\cdots M_q}\partial_{\alpha_{2n+2}}X^{M_2}\cdots\partial_{\alpha_p}X^{M_q}\right]\Big]\,.
\end{aligned}
\tag{2.19}
$$

In the same way we get that the equation of motion for A_α are

$$
\partial_\beta\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}^\alpha=0\,,
\tag{2.20}
$$

where

$$
\tilde{J}^{\alpha_1}=\sum_{n\geq 0}\frac{2n}{n!2^nq!}\epsilon^{\alpha_1\cdots\alpha_p}\partial_{\alpha_2}\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+2}\cdots\alpha_p}\right].
\tag{2.21}
$$

Let us again return to the ansatz (2.13) and (2.15) and calculate the matrix $\mathbf{A}_{\mu\nu}$

$$
\mathbf{A}=\begin{pmatrix}\tilde{\mathbf{a}}_{\alpha\beta}+a^2f'^2\partial_\alpha t\partial_\beta t & -a^2f'^2\partial_\beta t\\ -a^2f'^2\partial_\alpha t & a^2f'^2\end{pmatrix}
\tag{2.22}
$$

where

$$
\tilde{\mathbf{a}}_{\alpha\beta}=(g_{MN}+b_{MN})\partial_\alpha X^M\partial_\beta X^N+F_{\alpha\beta}\,.
\tag{2.23}
$$

Now using the fact that

$$
\det\mathbf{A}=\det\left(\mathbf{A}_{\alpha\beta}-\mathbf{A}_{\alpha x}\frac{1}{\mathbf{A}_{xx}}\mathbf{A}_{x\beta}\right)\det\mathbf{A}_{xx}
\tag{2.24}
$$

we get

$$
\det\mathbf{A}=a^2f'^2\det\tilde{\mathbf{a}}\,.
\tag{2.25}
$$

As a next step we determine the inverse matrix $(\mathbf{A}^{-1})$ up the correction $\mathcal{O}\left(\frac{1}{a}\right)$. After some algebra we get

$$
\begin{aligned}
(\mathbf{A}^{-1})^{\alpha\beta} &= (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\,, \qquad & (\mathbf{A}^{-1})^{x\beta}&=\partial_\alpha t(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\,,\\
(\mathbf{A}^{-1})^{\alpha x} &= (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t\,, \qquad & (\mathbf{A}^{-1})^{xx}&=\partial_\alpha t(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t
\end{aligned}
\tag{2.26}
$$

In the limit of large a. Using also the relation $\mathbf{A}_{\mu\nu}(\mathbf{A}^{-1})^{\nu\rho}=\delta_\mu^{\ \rho}$ and the form of the matrix $\mathbf{A}$ given in (2.22) we easily determine following relation

$$
(\mathbf{A}^{-1})^{\mu x}_S-(\mathbf{A}^{-1})^{\mu\alpha}_S\partial_\alpha t=\frac{1}{a^2f'^2}\left(\delta^\mu_x-(\mathbf{A}^{-1})^{x\mu}_S\right).
\tag{2.27}
$$

Now with the help of (2.27) we get

$$
\begin{aligned}
\partial_\mu\left[e^{-\Phi}V\partial_\nu T(\mathbf{A}^{-1})^{\nu\mu}_S\sqrt{-\det\mathbf{A}}\right] &= \partial_\mu\left[e^{-\Phi}Vaf'((\mathbf{A}^{-1})^{x\mu}_S-(\mathbf{A}^{-1})^{\alpha\mu}_S\partial_\alpha t)\sqrt{-\det\mathbf{A}}\right]\\
&= \partial_\mu\left[e^{-\Phi}V(\delta^\mu_x-(\mathbf{A}^{-1})^{x\mu}_S)\sqrt{-\det\tilde{\mathbf{a}}}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \partial_x \left[e^{-\Phi} V(T)(1 - (\mathbf{A}^{-1})^{xx}_S)\sqrt{-\det \tilde{\mathbf{a}}} \right] - \\
&\quad -\partial_\alpha \left[e^{-\Phi} V (\mathbf{A}^{-1})^{x\alpha}_S \sqrt{-\det \tilde{\mathbf{a}}} \right] \\
&= V' a f' e^{-\Phi} (1 - (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S \partial_\alpha t \partial_\beta t) \sqrt{-\det \tilde{\mathbf{a}}} + \\
&\quad + V' a f' \partial_\alpha t (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S \partial_\beta t \sqrt{-\det \tilde{\mathbf{a}}} - \\
&\quad - V \partial_\alpha \left[e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S \partial_\beta t \sqrt{-\det \tilde{\mathbf{a}}} \right] \qquad (2.28) \\
&= V' a f' e^{-\Phi} \sqrt{-\det \tilde{\mathbf{a}}} - V \partial_\alpha \left[e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S \partial_\beta t \sqrt{-\det \tilde{\mathbf{a}}} \right],
\end{aligned}
$$

where we have used the fact that $\partial_\alpha V = V' \partial_\alpha T = -aV' f' \partial_\alpha t$ and also the fact that the only field that depends on x is tachyon T. Using (2.28) we get following form of the DBI part of the tachyon equation of motion (2.4)

$$
V \partial_\alpha \left[e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S \partial_\beta t \sqrt{-\det \tilde{\mathbf{a}}} \right]. \tag{2.29}
$$

Now we consider the DBI part of the equation of motion for X^K (2.5). With the ansatz (2.13) and (2.15) the first two lines there take the form

$$
\begin{aligned}
&-af' V \partial_K [e^{-\Phi}] \sqrt{-\det \tilde{\mathbf{a}}} \\
&-af' V \frac{e^{-\Phi}}{2} \left(\partial_K g_{MN} + \partial_K b_{MN} \right) \partial_\alpha X^M \partial_\beta X^N (\tilde{\mathbf{a}}^{-1})^{\beta\alpha} \sqrt{-\det \tilde{\mathbf{a}}}.
\end{aligned} \tag{2.30}
$$

On the other hand the expression on the third line in (2.5) takes the form

$$
\partial_\mu \left[e^{-\Phi} V g_{KM} \partial_\alpha X^M (\mathbf{A}^{-1})^{\alpha\mu}_S a f' \sqrt{-\det \tilde{\mathbf{a}}} \right] = a V f' \partial_\beta \left[e^{-\Phi} g_{KM} \partial_\alpha X^M (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S \sqrt{-\det \tilde{\mathbf{a}}} \right], \tag{2.31}
$$

where we have used

$$
\partial_\beta [a V f'] = -\partial_x [a V f'] \partial_\beta t \tag{2.32}
$$

and also the fact that X^K are function of ξ^α only. In the same way as in (2.31) we can show that

$$
\partial_\mu \left[e^{-\Phi} V b_{KM} \partial_\alpha X^M (\mathbf{A}^{-1})^{\alpha\mu}_A a f' \sqrt{-\det \tilde{\mathbf{a}}} \right] = a f' V \partial_\beta \left[e^{-\Phi} b_{KM} \partial_\alpha X^M (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A \sqrt{-\det \tilde{\mathbf{a}}} \right]. \tag{2.33}
$$

If we collect all these results we obtain that the DBI part of the equation of motion for X^K takes the form

$$
\begin{aligned}
a f' V \Big(& -\partial_K [e^{-\Phi}] \sqrt{-\det \tilde{\mathbf{a}}} - \\
& - \frac{e^{-\Phi}}{2} \left(\partial_K g_{MN} + \partial_K b_{MN} \right) \partial_\alpha X^M \partial_\beta X^N (\tilde{\mathbf{a}}^{-1})^{\beta\alpha} \sqrt{-\det \tilde{\mathbf{a}}} + \\
& + \partial_\beta \left[e^{-\Phi} g_{KM} \partial_\alpha X^M (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S \sqrt{-\det \tilde{\mathbf{a}}} \right] + \\
& + \partial_\beta \left[e^{-\Phi} b_{KM} \partial_\alpha X^M (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A \sqrt{-\det \tilde{\mathbf{a}}} \right] \Big).
\end{aligned} \tag{2.34}
$$

Now let us consider the equation of motion for gauge field. For A_x we get

$$
\partial_\nu \left[V e^{-\Phi} (\mathbf{A}^{-1})^{x\nu}_A \sqrt{-\det \mathbf{A}} \right] = a f' V \partial_\beta t \partial_\alpha \left[e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A \sqrt{-\det \tilde{\mathbf{a}}} \right], \tag{2.35}
$$

where we have used an antisymmetry of $(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A$ so that $(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A \partial_\alpha \partial_\beta t = 0$.

On the other hand the equations of motion for A_α take the form

$$\partial_\mu \left[e^{-\Phi} V (\mathbf{A}^{-1})^{\alpha\mu}_A \sqrt{-\det(\mathbf{A}^{-1})} \right] = \partial_x [af'V] e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A \partial_\beta t \sqrt{-\det \tilde{\mathbf{a}}} + \partial_\beta \left[af'V e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A \sqrt{-\det \tilde{\mathbf{a}}} \right] = af'V \partial_\beta \left[e^{-\Phi} (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A \sqrt{-\det \tilde{\mathbf{a}}} \right] . \tag{2.36}$$

As a next step we evaluate the currents given in (2.10), (2.11) and (2.12) for the ansatz (2.13) and (2.15).

To begin with we determine the components of the embedding of various fields. It is easy to see that

$$\mathcal{F}_{x\alpha} = -\mathcal{F}_{\alpha x} = F_{x\alpha} + b_{MN} \partial_x X^M \partial_\alpha X^N = 0 \tag{2.37}$$

due to the fact that all worldvolume massless modes do not depend on x and also thanks to the fact that $A_x = 0$. Then the only nonzero components of $\mathcal{F}_{\mu\nu}$ are $\tilde{\mathcal{F}}_{\alpha\beta}$. For $C^{(n)}$ the situation is the same, namely any component that in the subscript contains x vanishes since

$$C_{\ldots x \ldots} = C_{\ldots M \ldots} \partial_x X^M = 0 \, . \tag{2.38}$$

Now we begin with the gauge current J^μ. Firstly, J^x is equal to

$$\begin{aligned} J^x &= \sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{x\alpha_2\alpha_3\cdots\alpha_p\alpha_1} \partial_{\alpha_2} \left[V(T) (\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \partial_{\alpha_1} T \right] \\ &= -\sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{x\alpha_2\alpha_3\cdots\alpha_p\alpha_1} \partial_{\alpha_1} [Vaf'] \left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \partial_{\alpha_1} t \right] - \\ &\quad - af'V \sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{x\alpha_2\alpha_3\cdots\alpha_p\alpha_1} \partial_{\alpha_2} \left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \partial_{\alpha_1} t \right] \\ &= af'V \partial_{\alpha_1} t \sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{\alpha_1\cdots\alpha_p x} \partial_{\alpha_2} \left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \right] \\ &= af'V \partial_{\alpha_1} t \tilde{J}^{\alpha_1} \, , \end{aligned} \tag{2.39}$$

where we have used the fact that $\partial_{\alpha_1}(Vf') = -\partial_x(Vf')\partial_{\alpha_1} t$ and then an antisymmetry of $\epsilon^{x\alpha_1\cdots\alpha_p}$ so that $\epsilon^{\alpha_1\alpha_2\cdots}\partial_{\alpha_1}\partial_{\alpha_2} t = 0$. Also the form of the current $\tilde{J}^\alpha$ was given in (2.21).[7]

On the other hand the current J^{α_1} is equal to

$$\begin{aligned} J^{\alpha_1} &= \sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{\alpha_1\alpha_2\cdots\alpha_p x} \partial_{\alpha_2} \left[V(T) (\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \partial_x T \right] + \\ &\quad + \sum_{n \geq 0} \frac{2n}{n! 2^n q!} \epsilon^{\alpha_1 x\alpha_3\cdots\alpha_p\alpha_2} \partial_x \left[V(T) (\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \partial_{\alpha_2} T \right] \\ &= \sum_{n \geq 0} af'V \frac{2n}{n! 2^n q!} \epsilon^{\alpha_1\alpha_2\cdots\alpha_p x} \partial_{\alpha_2} \left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}} \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \right] \\ &= af'V \tilde{J}^{\alpha_1} \end{aligned} \tag{2.40}$$

[7] Note that $\epsilon^{\alpha_1\cdots\alpha_p x} = \epsilon^{\alpha_1\cdots\alpha_p}$.

using the fact that $(\mathcal{F})^{n-1}_{\dots x}$ and $C_{\dots x \dots}$ are equal to zero. If we now combine (2.36) with (2.40) we get

$$af'V\left[\partial_\beta\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}^\alpha\right]=0\,. \tag{2.41}$$

Let us now analyse the behaviour of the term $af'V$ in the limit $a\to\infty$. Since by definition $f'(u)$ is finite for all u it remains to study the properties of the expression aV. Since $V\sim e^{-T}$ for $T\to\infty$ we have

$$\begin{aligned}\lim_{a\to\infty}aV(f(a(x-t(\xi)) &= (\text{for } \ x\neq t(\xi))\\ \lim_{a\to\infty}\frac{a}{e^{f(a(x-t(\xi)))}} &= \frac{1}{(x-t(\xi))f'}\lim_{a\to\infty}e^{-f(a(x-t(\xi)))}=0\,.\end{aligned} \tag{2.42}$$

We see that for $x\neq t(\xi)$ the expression aV goes to zero in the limit $a\to\infty$. On the other hand for $x=t(\xi)$ the potential $V(0)=\tau_p$ and hence in order to obey the equation of motion for A_α we find that the expression in the bracket in (2.41) should vanish. In fact, this expression is the same as the equation of motion for A_α given in (2.20)

On the other hand using (2.35) and (2.39) the equation of motion for A_x takes the form

$$af'V\left[\partial_\beta t\left(\partial_\alpha\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}^\beta\right)\right]=0 \tag{2.43}$$

that clearly holds using the fact that all modes obey the equation of motion (2.20).

Now we will analyse the current J_K. Looking on its form (2.12) it is clear that the expressions on the first and the second line are nonzero for $\mu_{p+1}=x$ only. On the other hand the expression on the third line can be nonzero for $\mu_{p+1}=x$ and for $\mu_1=x$ where we get

$$\begin{aligned}&-2n\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_1}\left[aVf'b_{KM}\partial_{\alpha_2}X^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]+\\ &\qquad+2n\epsilon^{x\alpha_2\cdots\alpha_p\alpha_1}\partial_x[aVf']\left[\partial_{\alpha_1}tb_{KM}\partial_{\alpha_2}X^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]=\\ &\qquad=-af'V2n\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}X^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]\,.\end{aligned} \tag{2.44}$$

Finally, the expression on the last line in (2.12) is equal to

$$\begin{aligned}&-q\epsilon^{\mu_1\cdots\mu_{p+1}}\partial_{\mu_{2n+1}}\left[V(T)(\mathcal{F})^n_{\mu_1\cdots\mu_{2n}}C_{KM_2\cdots M_q}\partial_{\mu_{2n+2}}X^{M_2}\cdots\partial_{\mu_p}X^{M_q}\partial_{\mu_{p+1}}T\right]=\\ &=-q\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_{2n+1}}\left[af'V(T)(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}C_{KM_2\cdots M_q}\partial_{\alpha_{2n+2}}X^{M_2}\cdots\partial_{\alpha_p}X^{M_q}\right]+\\ &\quad+q\epsilon^{\alpha_1\cdots\alpha_{2n}x\alpha_{2n+2}\cdots\alpha_p\alpha_{2n+1}}\partial_x\left[V(T)af'\partial_{\alpha_{2n+1}}t(\tilde{\mathcal{F}})^n_{\mu_1\cdots\mu_{2n}}C_{KM_2\cdots M_q}\partial_{\mu_{2n+2}}X^{M_2}\cdots\partial_{\mu_p}X^{M_q}\right]\\ &=-af'Vq\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_{2n+1}}\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}C_{KM_2\cdots M_q}\partial_{\alpha_{2n+2}}X^{M_2}\cdots\partial_{\alpha_p}X^{M_q}\right]\,.\end{aligned} \tag{2.45}$$

If we now combine all these results together we obtain final form of the current J_K

$$\begin{aligned}J_K=af'V\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}&\left(\partial_K b_{MN}\partial_{\alpha_1}X^M\partial_{\alpha_2}X^N(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}+\right.\\ &+(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\partial_K C_{M_1\cdots M_q}\partial_{\alpha_{2n+1}}X^{M_1}\cdots\partial_{\alpha_p}X^{M_q}-\\ &-2n\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}X^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]+\end{aligned}$$

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$$
+q\partial_{\alpha_{2n+1}}\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}C_{KM_2\cdots M_q}\partial_{\alpha_{2n+2}}X^{M_2}\cdots\partial_{\alpha_p}X^{M_q}\right]\Big)
$$

$$
\equiv af'V\tilde{J}_K\,, \tag{2.46}
$$

where $\tilde{J}_K$ was defined in (2.19). Using (2.34) and (2.46) we obtain the final form of the equation of motion for X^K in the form

$$
\begin{aligned}
af'V\Big(-\partial_K[e^{-\Phi}]\sqrt{-\det\tilde{\mathbf{a}}}-\frac{e^{-\Phi}}{2}\left(\partial_K g_{MN}+\partial_K b_{MN}\right)\partial_\alpha X^M\partial_\beta X^N(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\sqrt{-\det\tilde{\mathbf{a}}}+\\
+\partial_\beta\left[e^{-\Phi}g_{KM}\partial_\alpha X^M(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S\sqrt{-\det\tilde{\mathbf{a}}}\right]+\\
+\partial_\beta\left[e^{-\Phi}b_{KM}\partial_\alpha X^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}_K\Big)=0\,.
\end{aligned} \tag{2.47}
$$

Following discussion given below (2.41) we see that the expression in the bracket in (2.47) should be equal to zero. On the other hand this equation is exactly the equation of motion for the embedding mode that lives on the worldvolume of D(p-1)-brane that was given in (2.18).

Finally we come to the analysis of the tachyon current J_T that can be written as

$$
J_T=-\sum_{n\geq 0}V(T)\frac{1}{n!(2!)^n q!}\epsilon^{\mu_1\cdots\mu_{p+1}}\partial_{\mu_{p+1}}\left((\mathcal{F})^n_{\mu_1\cdots\mu_{2n}}C_{\mu_{2n+1}\cdots\mu_p}\right)\,. \tag{2.48}
$$

It is not hard to see that the tachyon current is equal to zero. Firstly, the contribution to J^T for which $\mu^{p+1}=x$ vanishes thanks to the fact that all massless modes do not depend on x. On the other hand for $\mu_{p+1}\neq x$ all contributions to J_T vanish since then there certainly exists $\mathcal{F}$ or C with the lower index containing x and as we argued above these terms are equal to zero. Hence we get

$$
J_T=0\,. \tag{2.49}
$$

Then the equation (2.4) takes the form

$$
V\partial_\alpha\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S\partial_\beta t\sqrt{-\det\tilde{\mathbf{a}}}\right]=0\,. \tag{2.50}
$$

Since for general background all massless fields depend on ξ the only way how to obey this equation for $x=t(\xi)$ where $V(0)=\tau_p$ is to demand that $\partial_\alpha t=0$. In other words we obtain a set of the tachyon kink solutions labelled with constant t that determines the position of the core of the kink on the worldvolume of an unstable Dp-brane. We mean that this is a natural result for a non-BPS Dp-brane where no gauge fixing procedure was imposed. In this case the position of a Dp-brane in the target spacetime is not specified and consequently all kink solutions on its worldvolume are equivalent.

In summary, we have shown that the spatial dependent tachyon condensation on the worldvolume of a unstable Dp-brane in general background leads to an emergence of a lower dimensional D(p-1)-brane where the massless modes that propagate on the worldvolume of the kink obey the equations of motion that arise from the DBI and WZ action for D(p-1)-brane.

2.1 Stress energy tensor

Further support for an interpretation of the tachyon kink as a lower dimensional D(p-1)-brane can be derived from the analysis of the stress energy tensor for the non-BPS Dp-brane. In order to find its form recall that we can write the action (2.1) as

$$S_p = -\int d^{10}x d^{(p+1)}\xi \delta(X^M(\xi) - x^M) e^{-\Phi} V(T)\sqrt{-\det \mathbf{A}}\,. \tag{2.51}$$

From (2.51) we can easily determine components of the stress energy tensor $T_{MN}(x)$ of an unstable D-brane using the fact that the stress energy tensor $T_{MN}(x)$ is defined as the variation of S_p with respect to $g_{MN}(x)$

$$\begin{aligned} T_{MN}(x) &= -2\frac{\delta S_p}{\sqrt{-g(x)}\delta g^{MN}(x)} \\ &= -\int d^{(p+1)}\xi \frac{\delta(X^M(\xi)-x^M)}{\sqrt{-g(x)}} e^{-\Phi} V g_{MK} g_{NL} \partial_\mu X^K \partial_\nu X^L (\mathbf{A}^{-1})^{\nu\mu}_S \sqrt{-\det \mathbf{A}}\,. \end{aligned} \tag{2.52}$$

Now from (2.13) and (2.15) we know that all massless modes are x independent. Hence (2.52) is equal to

$$\begin{aligned} T_{MN}(x) &= -\int dxaf' V(f(x)) \int d^p\xi \frac{\delta(X^M(\xi)-x^M)}{\sqrt{-g(x)}} \times \\ &\qquad \times e^{-\Phi} g_{MK} g_{NL} \partial_\alpha X^K \partial_\beta X^L (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S \sqrt{-\det \tilde{\mathbf{a}}} \\ &= -T_{p-1} \int d^p\xi \frac{\delta(X^M(\xi)-x^M)}{\sqrt{-g(x)}} e^{-\Phi} g_{MK} g_{NL} \partial_\alpha X^K \partial_\beta X^L (\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S \sqrt{-\det \tilde{\mathbf{a}}}\,, \end{aligned} \tag{2.53}$$

where

$$T_{p-1} = \int dxaV(f)f' = \int dmV(m) \tag{2.54}$$

is a tension of BPS D(p-1)-brane. In other words the stress energy tensor evaluated on the ansatz (2.13) and (2.15) corresponds to the stress energy tensor for D(p-1)-brane.

In the same way we can study other currents that express the coupling of the non-BPS Dp-brane to closed string massless fields. For example, let us consider current $J_C^{M_1\cdots M_N}$ corresponding to the variation of S_{WZ} with respect to $C_{M_1\cdots M_N}(x)$

$$\begin{aligned} J_C^{M_1\cdots M_N}(x) &= \frac{1}{n!(2!)^n N!} \int d^{p+1}\xi \delta^{10}(x^M - X^M(\xi)) V(T) \epsilon^{\mu_1\cdots\mu_{p+1}} \times \\ &\qquad \times (\mathcal{F})^n_{\mu_1\cdots\mu_{2n}} \partial_{\mu_{2n+1}} X^{M_1} \cdots \partial_{\mu_p} X^{M_N} \partial_{\mu_{p+1}} T\,, \end{aligned} \tag{2.55}$$

where $n = \frac{p-N}{2}$. It is clear that the nonzero components corresponds to $\mu_{p+1} = x$ (since in the opposite case there will be derivative $\partial_x X$ that for (2.15) vanishes) and we get

$$\begin{aligned} J_C^{M_1\cdots M_N}(x) &= \int dx V(f) f' a \frac{1}{n!(2!)^n N!} \int d^p\xi \delta^{10}(x^M - X^M(\xi)) \epsilon^{\alpha_1\cdots\alpha_p x} \times \\ &\qquad \times (\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}} \partial_{\alpha_{2n+1}} X^{M_1} \cdots \partial_{\alpha_p} X^{M_N} \\ &= \frac{\mu_{p-1}}{n!(2!)^n N!} \int d^p\xi \delta^{10}(x^M - X^M(\xi)) \epsilon^{\alpha_1\cdots\alpha_p x} \times \\ &\qquad \times (\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}} \partial_{\alpha_{2n+1}} X^{M_1} \cdots \partial_{\alpha_p} X^{M_N}\,, \end{aligned} \tag{2.56}$$

where $\mu_{p-1} = T_{p-1}$ is a Ramond-Ramond charge of D(p-1)-brane and hence (2.56) is an appropriate current for D(p-1)-brane.

3. Partial fixing gauge

In order to find solution of the tachyon effective action, where the mode t that determines the location of the core of the kink could be interpreted as an additional embedding coordinate, we should partial fix the gauge. In other words, when we choose one spatial coordinate on the worldvolume theory on which the tachyon depends we will also presume that this coordinate coincides with one arbitrary spatial coordinate in the target spacetime. Since both worldvolume theory and spacetime theory are diffeomorphism invariant we can without loose of generality choose the worldvolume direction on which the tachyon depends to be ξ^p and the spacetime direction to be X^9. Then we demand that

$$X^9 = x \equiv \xi^p \,. \tag{3.1}$$

Let us now consider following ansatz for the tachyon

$$T(x,\xi) = f(a(x-t(\xi))\,, \tag{3.2}$$

where $f(u)$ could be the same function as was defined in previous section. We also presume following ansatz for massless modes

$$X^I(x,\xi) = X^I(\xi)\,, \qquad A_x(x,\xi) = 0\,, \qquad A_\alpha(x,\xi) = A_\alpha(\xi)\,, \tag{3.3}$$

where $I,J,K = 0,1\dots,8$ and where $\xi^\alpha, \alpha = 0,\dots,p-1$ are coordinates tangential to the kink worldvolume.

Now we must show that the ansatz (3.2) and (3.3) solve the equation of motion for T, X^M and A_μ. Firstly, for (3.2) and (3.3) the matrix $\mathbf{A}_{\mu\nu}$ takes the form

$$\begin{aligned}
\mathbf{A}_{xx} &= g_{99} + a^2 f'^2, \mathbf{A}_{x\beta} = g_{9I}\partial_\beta X^I + b_{9I}\partial_\beta X^I - a^2 f'^2 \partial_\beta t \equiv H_{x\beta} - a^2 f'^2 \partial_\beta t\,, \\
\mathbf{A}_{\alpha x} &= \partial_\alpha X^I g_{I9} + \partial_\alpha X^I b_{I9} - a^2 f'^2 \partial_\alpha t \equiv H_{\alpha x} - a^2 f'^2 \partial_\alpha t\,, \\
\mathbf{A}_{\alpha\beta} &= (a^2 f'^2 - g_{99})\partial_\alpha t \partial_\beta t - H_{\alpha x}\partial_\beta t - \partial_\alpha t H_{x\beta} + \tilde{\mathbf{a}}_{\alpha\beta}\,, \\
\tilde{\mathbf{a}}_{\alpha\beta} &= g_{99}\partial_\alpha t\partial_\beta t + g_{IJ}\partial_\alpha X^I \partial_\beta X^J + \partial_\alpha X^I g_{I9}\partial_\beta t + \partial_\alpha t g_{9J}\partial_\beta X^J + \\
&\quad + b_{IJ}\partial_\alpha X^I \partial_\beta X^J + \partial_\alpha X^I b_{I9}\partial_\beta t + \partial_\alpha t b_{9J}\partial_\beta X^J + F_{\alpha\beta}\,.
\end{aligned} \tag{3.4}$$

As in the previous section we obtain that $\det \mathbf{A}$ is equal to

$$\det \mathbf{A} = a^2 f'^2 \det(\tilde{\mathbf{a}}_{\alpha\beta}) + O(1/a) \tag{3.5}$$

and the inverse matrix $(\mathbf{A}^{-1})$ when it is expressed as function of $(\tilde{\mathbf{a}}^{-1})$ and ∂t takes the form

$$\begin{aligned}
(\mathbf{A}^{-1})^{\alpha\beta} &= (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\,, \qquad & (\mathbf{A}^{-1})^{x\beta} &= \partial_\alpha t (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\,, \\
(\mathbf{A}^{-1})^{\alpha x} &= (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t\,, \qquad & (\mathbf{A}^{-1})^{xx} &= \partial_\alpha t(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t\,,
\end{aligned} \tag{3.6}$$

where the relations in (3.6) hold up to corrections of order $1/a^2$.

Now using the form of the matrix $\mathbf{A}$ (3.4) and the equation $(\mathbf{A}^{-1})^{\mu\nu}\mathbf{A}_{\nu\rho} = \delta^\mu_\rho$ we easily determine following exact relation

$$(\mathbf{A}^{-1})^{\mu x}_S - (\mathbf{A}^{-1})^{\mu\alpha}_S\partial_\alpha t = \frac{1}{a^2 f'^2}\left(\delta^\mu_x - (\mathbf{A}^{-1})^{xx}_S g_{99} - \frac{1}{2}\left((\mathbf{A}^{-1})^{\mu\alpha}H_{\alpha x} + H_{x\alpha}(\mathbf{A}^{-1})^{\alpha\mu}\right)\right). \tag{3.7}$$

Then with the help of (3.7) we can write the second term in (2.4) as

$$\partial_\mu\left[e^{-\Phi}V\sqrt{-\det\mathbf{A}}(\mathbf{A}^{-1})^{\mu\nu}_S\partial_\nu T\right] = \tag{3.8}$$
$$= \partial_\mu\left[e^{-\Phi}Vaf'\frac{1}{a^2f'^2}(\delta^\mu_x - (\mathbf{A}^{-1})^{xx}_S g_{99} - \frac{1}{2}((\mathbf{A}^{-1})^{\mu\alpha}H_{\alpha x} + H_{x\alpha}(\mathbf{A}^{-1})^{\alpha\mu}))\sqrt{-\det\mathbf{A}}\right]$$

Following [15] we can now argue that due to the explicit factor of $a^2 f'^2$ in the denominator the leading contribution from individual terms in this expression is now of order a and hence we can use the approximative results of $\det\mathbf{A}$ and $(\mathbf{A}^{-1})$ given in (3.5) and (3.6) to analyse the DBI part of the equation of motion for tachyon (2.4)

$$\begin{aligned}
&\partial_\mu\Big[e^{-\Phi}V\sqrt{-\det\mathbf{A}}af'\frac{1}{a^2f'^2}(\delta^\mu_x - (\mathbf{A}^{-1})^{\mu x}g_{99} - \\
&\quad -\frac{1}{2}(\mathbf{A}^{-1})^{\mu\alpha}H_{\alpha x} - \frac{1}{2}H_{x\alpha}(\mathbf{A}^{-1})^{\alpha\mu})\Big] - e^{-\Phi}V'\sqrt{-\det\mathbf{A}} = \\
&= \partial_x\Big[e^{-\Phi}V\sqrt{-\det\tilde{\mathbf{a}}}(1 - (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{99}\partial_\alpha t\partial_\beta t - \\
&\qquad -\frac{1}{2}\partial_\beta t(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}H_{\alpha x} - \frac{1}{2}H_{x\alpha}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t)\Big] - \\
&\qquad -\partial_\alpha\left[e^{-\Phi}V\sqrt{-\det\tilde{\mathbf{a}}}\left((\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{99}\partial_\beta t + \frac{1}{2}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}H_{\beta x} + \frac{1}{2}H_{x\alpha}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\right)\right] - \\
&-af'e^{-\Phi}V'\sqrt{-\det\tilde{\mathbf{a}}} \\
&= V\Big\{\partial_x\Big[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(1 - (\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{99}\partial_\alpha t\partial_\beta t - \frac{1}{2}H_{x\alpha}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t - \frac{1}{2}\partial_\alpha t(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}H_{\beta x})\Big] - \\
&\qquad -\partial_\alpha\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}((\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{99}\partial_\beta t + \frac{1}{2}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}H_{\beta x} + \frac{1}{2}H_{x\beta}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha})\right]\Big\}.
\end{aligned} \tag{3.9}$$

We should now more carefully interpret the result given above. Firstly, as we know from the previous section the tachyon potential V is equal to zero for $x - t(\xi) \neq 0$ while for $x - t(\xi) = 0$ we get $V(0) = \tau_p$ in the limit $a \to \infty$. Moreover, we will show in the next subsection that the tachyon current J_T is equal to $J_T = -V\tilde{J}_9$ when it is evaluated on the ansatz (3.2) and (3.3). Note that $\tilde{J}_9$ is gauge fixed version of the current (2.19). The main point is that the tachyon equation of motion is obeyed for $x - t(\xi) \neq 0$ while for $x = t(\xi)$ we should demand that the expression in the bracket in (3.9) together with $-\tilde{J}_9$ should in be equal to zero. If we now use the fact that

$$\begin{aligned}
H_{x\alpha}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}\partial_\beta t + \partial_\beta t(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}H_{\alpha x} &= 2(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{I9}\partial_\alpha X^I\partial_\beta t - 2(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A b_{I9}\partial_\alpha X^I\partial_\beta t\,, \\
(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}H_{\beta x} + H_{x\beta}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha} &= 2(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{I9}\partial_\beta X^I - 2(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A b_{9I}\partial_\beta X^I
\end{aligned} \tag{3.10}$$

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we can write the expression in the bracket in (3.9) with $-\tilde{J}_9$ in the form

$$\begin{aligned}
&\frac{\delta e^{-\Phi}}{\delta x}\sqrt{-\det\tilde{\mathbf{a}}}+\frac{e^{-\Phi}}{2}\left(\frac{\delta g_{MN}}{\delta x}\partial_\alpha Y^M\partial_\beta Y^N+\frac{\delta b_{MN}}{\delta x}\partial_\alpha Y^M\partial_\beta Y^N\right)(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}\sqrt{-\det\tilde{\mathbf{a}}}-\\
&\qquad-\partial_\alpha\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{9M}\partial_\beta Y^M\right]-\\
&\qquad-\partial_x\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S g_{9M}\right]\partial_\alpha t\partial_\beta Y^M-\\
&\qquad-\partial_\alpha\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A b_{9M}\partial_\beta Y^M\right]-\\
&\qquad-\partial_9\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A b_{9M}\right]\partial_\alpha Y^M\partial_\beta t-\tilde{J}_9=0\,,
\end{aligned}\tag{3.11}$$

where we have introduced the notation

$$Y^M\,,\qquad M=0,\dots,9\,,\qquad Y^I=X^I\,,\qquad I=0,\dots,8\,,\qquad Y^9=t\,.\tag{3.12}$$

It is important to stress that in (3.11) we firstly perform the derivative with respect to x and then we replace x with $t(\xi)$. Then the presence of the following expressions in (3.9)

$$-\partial_x\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}g_{9M}\right]\partial_\alpha t\partial_\beta Y^M-\partial_x\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A b_{9M}\right]\partial_\alpha Y^M\partial_\beta t\tag{3.13}$$

is crucial for an interpretation of $t(\xi)$ as an additional scalar field that parametrises the position of D(p-1)-brane in x direction.

To see this more clearly let us compare (3.11) with the equation of motion (2.18) for $K=9$ and observe that the expression on the third line in (2.18) can be written as

$$\begin{aligned}
&\partial_\alpha\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}g_{9M}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S\right]=\\
&\qquad=\partial_\alpha\left[e^{-\Phi(\xi,x)}\sqrt{-\det\tilde{\mathbf{a}}(\xi,x)}g_{9M}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S(\xi,x)\right]\\
&\qquad+\partial_x\left[e^{-\Phi(\xi,x)}\sqrt{-\det\tilde{\mathbf{a}}_{BPS}(\xi,x)}g_{9M}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S(\xi,x)\right]\partial_\alpha Y\partial_\beta Y^M\,,
\end{aligned}\tag{3.14}$$

where on the second line the derivative with respect to ξ^α treats x as an independent variable so that we firstly perform derivative with respect to ξ^α and then we replace x with Y. We see that this prescription coincides with the expressions on the second line in (3.11). In the same way we can proceed with the expression on the fourth line in (2.18)

$$\begin{aligned}
&-\partial_\alpha\left[e^{-\Phi}\sqrt{-\det\tilde{\mathbf{a}}}b_{9M}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\right]=\\
&\qquad\partial_\alpha\left[e^{-\Phi(\xi,x)}\sqrt{-\det\tilde{\mathbf{a}}(\xi,x)}b_{9M}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A(\xi,x)\right]\\
&\qquad+\partial_x\left[e^{-\Phi(\xi,x)}\sqrt{-\det\tilde{\mathbf{a}}(\xi,x)}b_{9M}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A(\xi,x)\right]\partial_\alpha Y\partial_\beta Y^M
\end{aligned}\tag{3.15}$$

and this again coincides with the expressions on the fourth line in (3.11). In summary, the location of the tachyon kink in the x^9 direction is completely determined by field $t(\xi)$ that obeys the equation of motion (2.18) for $K=9$.

Now we come to the analysis of the equation of motion for $X^K\,, K=0,\dots,8$. For the ansatz (3.2) and (3.3) the first term in (2.5) takes the form

$$\frac{\delta e^{-\Phi}}{\delta X^K}V\sqrt{-\det\mathbf{A}}=af'V\frac{\delta e^{-\Phi}}{\delta X^K}\sqrt{-\det\tilde{\mathbf{a}}}\,. \tag{3.16}$$

On the other hand the expression on the second line in (2.5) can be written as

$$\begin{aligned}e^{-\Phi}V\left[\frac{\delta g_{MN}}{\delta X^K}\partial_\mu X^M\partial_\nu X^N+\frac{\delta b_{MN}}{\delta X^K}\partial_\mu X^M\partial_\nu X^N\right](\mathbf{A}^{-1})^{\nu\mu}\sqrt{-\det\mathbf{A}}= \\ af'Ve^{-\Phi}\left[\frac{\delta g_{MN}}{\delta Y^K}\partial_\alpha Y^M\partial_\beta Y^N+\frac{\delta b_{MN}}{\delta Y^K}\partial_\alpha Y^M\partial_\beta Y^N\right](\tilde{\mathbf{a}}^{-1})^{\beta\alpha}\sqrt{-\det\tilde{\mathbf{a}}}\,,\end{aligned} \tag{3.17}$$

where we have used the notation (3.12). Finally we will analyse the expression on the third and the fourth line in (2.5) that can be written as

$$\partial_\mu\left[e^{-\Phi}V(g_{KM}\partial_\nu X^M(\mathbf{A}^{-1})^{\nu\mu}_S+b_{KM}\partial_\nu X^M(\mathbf{A}^{-1})^{\nu\mu}_A)\sqrt{-\det\mathbf{A}}\right]. \tag{3.18}$$

After some lengthy calculations we obtain that (3.18) for the ansatz (3.2) and (3.3) takes the form

$$\begin{aligned}aVf'\Big(\partial_x\left[e^{-\Phi}(g_{KM}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S+b_{KM}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A)\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\alpha Y^M\partial_\beta t+ \\ +\partial_\alpha\left[e^{-\Phi}(g_{KM}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S+b_{KM}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A)\sqrt{-\det\tilde{\mathbf{a}}}\right]\Big).\end{aligned} \tag{3.19}$$

Finally, using (3.16), (3.17) and (3.19) we get

$$\begin{aligned}af'V\Big\{&-\frac{\delta e^{-\Phi}}{\delta X^K}\sqrt{-\det\tilde{\mathbf{a}}}- \\ &-\frac{e^{-\Phi}}{2}\left[\frac{\delta g_{MN}}{\delta Y^K}\partial_\alpha Y^M\partial_\beta Y^N+\frac{b_{MN}}{\delta Y^K}\partial_\alpha Y^M\partial_\beta Y^N\right](\tilde{\mathbf{a}}^{-1})^{\beta\alpha}\sqrt{-\det\tilde{\mathbf{a}}}+ \\ &+\partial_x\left[e^{-\Phi}(g_{KM}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S+b_{KM}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A)\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\alpha Y^M\partial_\beta t+ \\ &+\partial_\alpha\left[e^{-\Phi}(g_{KM}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_S+b_{KM}\partial_\beta Y^M(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A)\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}_K\Big\}=0\end{aligned} \tag{3.20}$$

using the result that will be proven in the next subsection that the current J_K is equal to $aVf'\tilde{J}_K$, where $\tilde{J}_K$ is given in (2.19).

As we know from the previous section the expression $af'V$ goes to zero in the limit $a\to\infty$ when $x\neq t(\xi)$. On the other hand for $x=t(\xi)$ the potential $V(0)=\tau_p$ for arbitrary a and hence in order to obey the equation of motion for X^K (2.5) we get that the expression in the bracket $\{\dots\}$ should vanish for $x=t(\xi)$. However this is precisely the equation of motion (2.18) and hence we again obtain the result that the scalar modes X^K should solve the equation of motion that arise from the action for BPS D(p-1)-brane.

Since we mean that it is very important to find the correct interpretation of the equation (3.20) we we would like again stress that in the expression in the bracket in (3.20) we firstly perform a derivative with respect to ξ^α and then we replace x with $t(\xi)$ in the limit

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$a \to \infty$. This fact implies that $t(\xi)$ is an scalar mode that parametrises the location of D(p-1)-brane in the x^9 direction.

To complete the discussion of the equation of motion for X^K we should also analyse the equation of motion for X^9. If we proceed in the same way as for X^K that was analysed above we obtain that the equation of motion for X^9 takes the form

$$\begin{aligned} af'V\Big(&-\frac{e^{-\Phi}}{2}\sqrt{-\det\tilde{\mathbf{a}}}\,[\partial_x g_{MN}+\partial_x b_{MN}]\,\partial_\alpha Y^M\partial_\beta Y^N(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}-\partial_x[e^{-\Phi}]\sqrt{-\det\tilde{\mathbf{a}}}+ \\ &+\partial_x\left[e^{-\Phi}g_{9M}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\alpha t\partial_\beta X^M+ \\ &+\partial_\beta\left[e^{-\Phi}g_{9M}\partial_\alpha Y^M(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_S\sqrt{-\det\tilde{\mathbf{a}}}\right]+ \\ &+\partial_x\left[e^{-\Phi}b_{9M}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\alpha t\partial_\beta X^M+ \\ &+\partial_\beta\left[e^{-\Phi}b_{9M}\partial_\alpha X^M(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}_9\Big)=0, \end{aligned} \tag{3.21}$$

where we have again used the result from the next subsection that $J_9 = afV'\tilde{J}_9$. We see that the expression in the bracket $(\dots)$ in (3.21) coincides with the equation of motion (2.18) for $K = 9$. This is nice result since we should obtain ten independent equations for scalar modes and we see that the equation of motion for T and for X^9 imply one equation of motion for mode t.

Finally we come to the analysis of the equation of motion for A_μ given in (2.6). For $\mu = \alpha$ the DBI part of the equation of motion (2.6) takes the form

$$\begin{aligned} &\partial_x[Vaf']e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\partial_\beta t\sqrt{-\det\tilde{\mathbf{a}}}+\partial_\beta[Vaf']e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}+ \\ &\quad Vaf'\left(\partial_x\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\beta t+\partial_\beta\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]\right). \end{aligned} \tag{3.22}$$

Again using (2.32) we see that the expressions on the first line cancel. If we now combine (3.22) with (3.37) we obtain final form of the equation of motion for A_α

$$Vaf'\left(\partial_x\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\beta t+\partial_\beta\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}^\alpha\right)=0\,. \tag{3.23}$$

As usual we demand that the expression in the bracket $(\dots)$ in (3.23) should be equal to zero for $x = t(\xi)$. Then the vanishing of this expression is equivalent to

$$\partial_\alpha\left[e^{-\Phi(t(\xi))}\sqrt{-\det\tilde{\mathbf{a}}(t(\xi))}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A(t(\xi))\right]+\tilde{J}^\alpha(t(\xi))=0 \tag{3.24}$$

that is an equation of motion for the gauge field given in (2.20).

Finally, the DBI part of the equation of motion (2.6) for $\mu = x$ and for the ansatz (3.2), (3.3) takes the form

$$\begin{aligned} \partial_\nu\left[Ve^{-\Phi}(\mathbf{A}^{-1})^{x\nu}_A\sqrt{-\det\mathbf{A}}\right] &= \partial_\beta\left[Vaf'e^{-\Phi}\partial_\alpha t(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right] \\ &= Vaf'\partial_\beta t\partial_\alpha\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\sqrt{-\det\tilde{\mathbf{a}}}\right] \end{aligned} \tag{3.25}$$

using (2.32) and then an antisymmetry of the matrix $(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A$. Now with the help of the current J^x given in (3.39) and with (3.25) the equation of motion (2.6) for $\mu = x$ takes the form

$$\begin{aligned} &aVf'\left\{\partial_\beta t\left(\partial_\alpha\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\tilde{J}^\beta\right)\right\}= \qquad (3.26)\\ &=aVf'\left\{\partial_\beta t\left(\partial_\alpha\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\beta\alpha}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]+\partial_x\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]\partial_\alpha t+\tilde{J}^\beta\right)\right\}=0\,, \end{aligned}$$

where we have included an expression $aVf'\partial_\beta t\partial_\alpha t\partial_x\left[e^{-\Phi}(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A\sqrt{-\det\tilde{\mathbf{a}}}\right]$ that vanishes thanks to the antisymmetry of $(\tilde{\mathbf{a}}^{-1})^{\alpha\beta}_A$ however whose presence is crucial for an interpretation of t as an embedding coordinate. Following arguments given above we obtain that the expression in the bracket $\{\dots\}$ should be equal to zero for $x = t(\xi)$ in the limit $a \to \infty$. We see that this holds since as we have argued above the massless modes obey (2.20).

In summary, we have shown that the dynamics of the tachyon kink is governed by the equation of motion that arises from the DBI and WZ action for D(p-1)-brane that is localised at the point $x = t(\xi)$. To really conclude this section we should now evaluate currents J_M, J^μ and J^T.

3.1 Analysis of currents

In this subsection we will analyse the currents (2.10) , (2.11) and (2.12) for the ansatz given in (3.2) and (3.3). We will see that this analysis is much more difficult that in the case when we did not impose any gauge fixing conditions.

We start with the gauge current (2.10) where $\mu_1 = \alpha_1$. In this case we get

$$\begin{aligned} J^{\alpha_1} &= \sum_{n\geq 0}\frac{2n}{n!2^nq!}\epsilon^{\alpha_1\mu_1\cdots\mu_{p+1}}\partial_{\mu_2}\left[V(T)(\mathcal{F})^{n-1}_{\mu_3\cdots\mu_{2n}}C_{\mu_{2n+1}\cdots\mu_p}\partial_{\mu_{p+1}}T\right]=\\ &=\sum_{n\geq 0}\frac{2n}{n!2^nq!}\epsilon^{\alpha_1\alpha_2\cdots\alpha_px}\partial_{\alpha_2}\left[V(T)(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\partial_xT\right]+\\ &\quad+\sum_{n\geq 0}\frac{2n}{n!2^nq!}\epsilon^{\alpha_1x\alpha_2\cdots\alpha_p}\partial_x\left[V(T)(\mathcal{F})^{n-1}_{\alpha_2\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_{p-1}}\partial_{\alpha_p}T\right]+\\ &\quad+\sum_{n\geq 0}\frac{4n(n-1)}{n!2^nq!}\epsilon^{\alpha_1\alpha_2x\alpha_4\cdots\alpha_p\alpha_3}\partial_{\alpha_2}\left[V(T)\mathcal{F}_{x\alpha_4}(\mathcal{F})^{n-2}_{\alpha_5\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\partial_{\alpha_3}T\right]+ \qquad (3.27)\\ &\quad+\sum_{n\geq 0}\frac{2nq}{n!2^nq!}\epsilon^{\alpha_1\cdots\alpha_{2n}x\alpha_{2n+2}\cdots\alpha_p\alpha_{2n+1}}\partial_{\alpha_2}\left[V(T)(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{x\alpha_{2n+2}\cdots\alpha_p}\partial_{\alpha_{2n+1}}T\right]. \end{aligned}$$

It can be shown that for the ansatz (3.2) the expressions on the second and the third line in (3.27) take the form

$$\begin{aligned} &af'V\sum_{n\geq 0}\frac{2n}{n!2^nq!}\epsilon^{\alpha_1\cdots\alpha_px}\partial_{\alpha_2}\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]+\\ &\qquad+aVf'\sum_{n\geq 0}\frac{2n}{n!2^nq!}\epsilon^{\alpha_1\cdots\alpha_px}\partial_x\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]\partial_{\alpha_2}t\,. \qquad (3.28) \end{aligned}$$

Now we come to one important point. As we know from the previous section the factor aVf' vanishes for $x \neq t(\xi)$ for $a \to \infty$. At the same time we argued that we should regard $t(\xi)$ as an embedding coordinate. On the other hand $\mathcal{F}_{\alpha\beta}$ contains an embedding of B that is equal to

$$B_{IJ}\partial_\alpha X^I \partial_\beta X^J \tag{3.29}$$

and we also have

$$C_{\alpha_{2n+1}\cdots\alpha_p} = C_{I_{2n+1}\cdots I_p}\partial_{\alpha_{2n+1}}X^{I_{2n+1}}\cdots\partial_{\alpha_p}X^{I_p}\,. \tag{3.30}$$

Now we would like to argue that whenever some term in any current will contain a factor $\partial_{\alpha_x} t$ we can replace all $\mathcal{F}_{\alpha\beta}$ and all $C_{\alpha_{2n+1}\cdots\alpha_p}$ with $\tilde{\mathcal{F}}_{\alpha\beta}$ and $\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}$ where

$$\begin{aligned}\tilde{\mathcal{F}}_{\alpha\beta} &= \mathcal{F}_{\alpha\beta} + B_{MN}\partial_\alpha Y^M \partial_\beta Y^N\,,\\ \tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} &= C_{M_{2n+1}\cdots M_p}\partial_{\alpha_{2n+1}}Y^{M_{2n+1}}\cdots\partial_{\alpha_p}Y^{M_p}\,,\end{aligned} \tag{3.31}$$

where Y^M was introduced in (3.12). To see that this replacement is correct note that the additional terms in expressions (We mean expressions with the overall multiplicative factor $\partial_{\alpha_x} t$) , when we replace $\mathcal{F}$ with $\tilde{\mathcal{F}}$ and C with $\tilde{C}$ contain derivative of t in the form $\partial_{\alpha_y} t$. Now thanks to the existence of the factor $\epsilon^{\alpha_1\cdots\alpha_p x}$ it is clear that these terms after multiplication with $\partial_{\alpha_x} t$ vanish since

$$\epsilon^{\alpha_1\cdots\alpha_x\cdots\alpha_y\cdots\alpha_p x}\partial_{\alpha_x} t \partial_{\alpha_y} t = 0\,. \tag{3.32}$$

Now we proceed to the analysis of the expression on the fourth line in (3.27)

$$\begin{aligned}&\sum_{n\geq 0}\frac{4n(n-1)}{n!2^n q!}\epsilon^{\alpha_1\alpha_2 x\alpha_4\cdots\alpha_p\alpha_3}\partial_{\alpha_2}\left[V(T)\mathcal{F}_{x\alpha_4}(\mathcal{F})^{n-2}_{\alpha_5\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\partial_{\alpha_3}T\right] =\\ &\quad = aVf'\sum_{n\geq 0}\frac{4n(n-1)}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[b_{9I}\partial_{\alpha_3} t\partial_{\alpha_4}X^I(\mathcal{F})^{n-2}_{\alpha_5\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]\end{aligned} \tag{3.33}$$

using the fact that

$$\mathcal{F}_{x\alpha} = F_{x\alpha} + b_{9I}\partial_\alpha X^I = b_{9I}\partial_\alpha X^I\,. \tag{3.34}$$

Now it is easy to see that (3.33) together with (3.28) gives

$$\begin{aligned}&af'V\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right] +\\ &+aVf'\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]\partial_{\alpha_2} t +\\ &+aVf'\sum_{n\geq 0}\frac{4n(n-1)}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[b_{9I}\partial_{\alpha_3} t\partial_{\alpha_4}X^I(\mathcal{F})^{n-2}_{\alpha_5\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right] =\\ &\qquad\qquad = af'V\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right] +\\ &\qquad\qquad + aVf'\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]\partial_{\alpha_2} t\,.\end{aligned} \tag{3.35}$$

To complete the discussion of the current we should analyse the expression on the last line in (3.27)

$$\sum_{n\geq 0}\frac{2nq}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_{2n}x\alpha_{2n+2}\cdots\alpha_p\alpha_{2n+1}}\partial_{\alpha_2}\left[V(T)(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{x\alpha_{2n+2}\cdots\alpha_p}\partial_{\alpha_{2n+1}}T\right]=$$
$$=aVf'\sum_{n\geq 0}\frac{2nq}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{x\alpha_{2n+2}\cdots\alpha_p}\partial_{\alpha_{2n+1}}t\right]. \qquad (3.36)$$

Now we see that (3.36) is precisely the expression that is needed to replace $C_{\alpha_{2n+1}\cdots\alpha_p}$ with $\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}$ in (3.27). Finally, if we combine (3.35) with (3.36) we obtain following form of the current J^{α_1}

$$J^{\alpha_1}=af'V\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\alpha_2\cdots\alpha_p x}\partial_{\alpha_2}\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]+$$
$$+aVf'\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]\partial_{\alpha_2}t$$
$$=afV\tilde{J}^{\alpha_1}, \qquad (3.37)$$

where $\tilde{J}^{\alpha_1}$ is a gauge field current for D(p-1)-brane given in (2.21). Note also that the term on the second line in (3.37) is exactly the right one in order to interpret t as an embedding coordinate since in the expression on the first line in (3.37) the partial derivative ∂_{α_2} treats x as an independent variable. We will also see that in all other currents similar additional terms appear as well.

Finally, we will analyse the gauge current for $\mu_1=x$

$$J^x=\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{x\alpha_2\alpha_3\cdots\alpha_p\alpha_1}\partial_{\alpha_2}\left[V(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\partial_{\alpha_1}T\right]=$$
$$=af'V\partial_{\alpha_1}t\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right], \qquad (3.38)$$

where we have used an antisymmetry of $\epsilon^{x\alpha_1\cdots\alpha_p}$ under exchange of α_1 and α_p so that $\epsilon^{\alpha_1\cdots\alpha_p}\partial_{\alpha_1}\partial_{\alpha_p}t=0$.

Thanks to the presence of the term $\partial_{\alpha_1}t$ we can, following discussion given above, everywhere replace $\mathcal{F}$ with $\tilde{\mathcal{F}}$ and C with $\tilde{C}$. From the same reason we can add to (3.38) an expression $aVf'\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]\partial_{\alpha_2}t\partial_{\alpha_1}t$ that formally vanishes however with this term the current (3.38) can be written as

$$J^x=af'V\partial_{\alpha_1}t\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_2}\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]+$$
$$+aVf'\sum_{n\geq 0}\frac{2n}{n!2^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]\partial_{\alpha_2}t\partial_{\alpha_1}t$$
$$=aVf'\partial_{\alpha_1}t\tilde{J}^{\alpha_1}. \qquad (3.39)$$

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Let us now proceed to the analysis of the tachyon current (2.11) for the ansatz (3.2) and (3.3)

$$J_T = -V(T)\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\mu_1\cdots\mu_{p+1}}\partial_{\mu_{p+1}}\left[(\mathcal{F})^n_{\mu_1\cdots\mu_{2n}}C_{\mu_{2n+1}\cdots\mu_p}\right]. \tag{3.40}$$

Now we split the calculations into two parts, the first one when $\mu_{p+1}=x$ and the second one when $\mu_{p+1}\neq x$. In the first case we get

$$\begin{aligned}
&-V(T)\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x\left[(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]= \qquad (3.41)\\
&\qquad = -V\sum_{n\geq 0}\frac{n}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x b_{IJ}\partial_{\alpha_1}X^I\partial_{\alpha_2}X^J\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]-\\
&\qquad\quad -V\sum_{n\geq 0}\frac{q}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\left[(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}\partial_x C_{I_{2n+1}\cdots I_p}\partial_{\alpha_{2n+1}}X^{I_{2n+1}}\cdots\partial_{\alpha_p}X^{I_p}\right].
\end{aligned}$$

Now we extend the expression $\partial_x b_{IJ}\partial_{\alpha_1}X^I\partial_{\alpha_2}X^J$ as

$$\partial_x b_{IJ}\partial_{\alpha_1}X^I\partial_{\alpha_2}X^J+\partial_x b_{9J}\partial_{\alpha_1}t\partial_{\alpha_2}X^J+\partial_x b_{I9}\partial_{\alpha_1}X^I\partial_{\alpha_2}t-(\partial_x b_{9J}\partial_{\alpha_1}t\partial_{\alpha_2}X^J+\partial_x b_{I9}\partial_{\alpha_1}X^I\partial_{\alpha_2}t) \tag{3.42}$$

In the same way we can proceed with the expression $\partial_x C_{I_{2n+1}\cdots I_p}$. Then the expression (3.41) takes the form

$$\begin{aligned}
&-V\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_x b_{MN}\partial_{\alpha_1}Y^M\partial_{\alpha_2}Y^N\left[(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]-\\
&\qquad -V\sum_{n\geq 0}\frac{q}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\left[(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}\partial_x C_{M_{2n+1}\cdots M_p}\partial_{\alpha_{2n+1}}Y^{M_{2n+1}}\cdots\partial_{\alpha_p}Y^{M_p}\right]+\\
&\qquad +V\sum_{n\geq 0}\frac{2n}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}[\partial_x b_{9N}\partial_{\alpha_1}t\partial_{\alpha_2}Y^N](\tilde{\mathcal{F}})^{n-1}_{\alpha_1\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}+ \qquad (3.43)\\
&\qquad +V\sum_{n\geq 0}\frac{q}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}[\partial_x C_{9I_{2n+2}\cdots I_p}\partial_{\alpha_{2n+1}}t\partial_{\alpha_{2n+2}}Y^I\cdots\partial_{\alpha_p}Y^{I_p}],
\end{aligned}$$

where we have included tilde components defined in (3.31). We have also used the fact that we can write $b_{9I}\partial_{\alpha_2}X^I=b_{9M}\partial_\alpha Y^M$ and in the same way we can extend the embedding $C_{9I_{2n+2}\cdots I_p}\partial_{\alpha_{2n+2}}X^{I_{2n+2}}\cdots\partial_{\alpha_p}X^{I_p}$ to $C_{9M_{2n+2}\cdots M_p}\partial_{\alpha_{2n+2}}Y^{M_{2n+2}}\cdots\partial_{\alpha_p}Y^{M_p}$ using antisymmetry of $C_{M_1\cdots M_q}$.

Let us now consider the case when $\mu_{p+1}\neq x$ in (3.40). In this case we get

$$\begin{aligned}
&-V(T)\sum_{n\geq 0}\frac{2n}{n!(2!)^n q!}\epsilon^{x\alpha_2\cdots\alpha_p\alpha_1}\partial_{\alpha_1}\left[\mathcal{F}_{x\alpha_2}(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]-\\
&-V(T)\sum_{n\geq 0}\frac{q}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_{2n}x\alpha_{2n+2}\alpha_p\alpha_{2n+1}}\partial_{\alpha_{2n+1}}\left[(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}C_{x\alpha_{2n+2}\cdots\alpha_p}\right]=\\
&\qquad = V(T)\sum_{n\geq 0}\frac{2n}{n!(2!)^n q!}\epsilon^{\alpha_1\alpha_2\cdots\alpha_p x}\partial_{\alpha_1}\left[b_{9I}\partial_{\alpha_2}X^I(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]+ \qquad (3.44)\\
&\qquad\quad + V(T)\sum_{n\geq 0}\frac{q}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_{2n+1}}\left[(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}C_{9I_{2n+2}\cdots I_p}\partial_{\alpha_{2n+2}}X^{I_{2n+2}}\cdots\partial_{\alpha_p}X^{I_p}\right]
\end{aligned}$$

using the fact that $F_{x\alpha_1}=F_{\alpha_1 x}=0$.

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We will again argue that terms written on the third and the fourth line in (3.43) are important for an interpretation of t as an embedding coordinate. In fact, following discussion performed in previous section it is easy to see that

$$\partial_{\alpha_1}[b_{9I}\partial_{\alpha_2}X^I] = \partial_{\alpha_1}(b_{9I}(x,X))\partial_{\alpha_2}X^I\big|_{x=t(\xi)} + \partial_x(b_{9I}(x,X))\partial_{\alpha_2}X^I\big|_{x=t(\xi)}\partial_{\alpha_1}t + \\ +b_{9I}\partial_{\alpha_1}\partial_{\alpha_2}X^I\,, \tag{3.45}$$

where the second term vanishes after multiplying this derivative with $\epsilon^{\alpha_1\alpha_2\cdots}$. In the same way we can show that the derivative $\partial_{\alpha_2}\mathcal{F}_{\alpha_3\alpha_4}$ takes the form

$$\partial_{\alpha_2}F_{\alpha_3\alpha_4} + \partial_x b_{IJ}\partial_{\alpha_3}X^I\partial_{\alpha_4}X^J\partial_{\alpha_2}t\big|_{x=t(\xi)} + \\ + \partial_{\alpha_2}b_{IJ}(x,X)\partial_{\alpha_3}X^I\partial_{\alpha_4}X^J\big|_{x=t(\xi)} + b_{IJ}(x,X)\partial_{\alpha_2}\left(\partial_{\alpha_3}X^I\partial_{\alpha_4}X^J\right). \tag{3.46}$$

If we multiply the expression given above with $\epsilon^{\alpha_2\alpha_3\alpha_4\cdots}$ we obtain that the first and the last term vanishes as can be seen from following examples

$$\epsilon^{\alpha_2\alpha_3\alpha_4}\partial_{\alpha_2}\partial_{\alpha_3}A_{\alpha_4} = -\epsilon^{\alpha_3\alpha_2\alpha_4}\partial_{\alpha_3}\partial_{\alpha_2}A_{\alpha_4}\,, \\ \epsilon^{\alpha_2\alpha_3\alpha_4}\partial_{\alpha_2}\partial_{\alpha_3}X^I = -\epsilon^{\alpha_3\alpha_2\alpha_4}\partial_{\alpha_3}\partial_{\alpha_2}X^I\,. \tag{3.47}$$

If we now combine (3.43) with (3.44) we obtain that the tachyon current has natural interpretation as the current for the scalar mode $t(\xi)$ that parametrises location of D(p-1)-brane in the x^9 direction

$$J_T = -V\tilde{J}_9 \tag{3.48}$$

with $\tilde{J}_9$ given in (2.21).

Finally we will analyse currents J_K given in (2.12). Let us start with the first term in (2.12)

$$\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\mu_1\cdots\mu_{p+1}}V(T)\partial_K b_{MN}\partial_{\mu_1}X^M\partial_{\mu_2}X^N(\mathcal{F})^{n-1}_{\mu_3\cdots\mu_{2n}}C_{\mu_{2n+1}\cdots\mu_p}\partial_{\mu_{p+1}}T = \\ = aVf'\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\Big(\partial_K b_{MN}\partial_{\alpha_1}Y^I\partial_{\alpha_2}Y^N(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p} + \\ +2(n-1)\partial_K b_{MN}\partial_{\alpha_1}Y^M\partial_{\alpha_2}Y^N b_{9\alpha_4}\partial_{\alpha_3}t(\mathcal{F})^{n-2}_{\alpha_5\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p} \\ + q\partial_K b_{MN}\partial_{\alpha_1}Y^M\partial_{\alpha_2}Y^N(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{x\alpha_{2n+2}\cdots\alpha_p}\partial_{\alpha_{2n+1}}t\Big). \tag{3.49}$$

In the previous expressions we have included the terms with tilde from the same reasons as was argued in case of gauge field current. We can also simplify the expression above using the fact that

$$\epsilon^{\alpha_1\alpha_2\alpha_3\cdots\alpha_{2n}\cdots}\left(\mathcal{F}^{n-1}_{\alpha_3\cdots\alpha_{2n}} + 2(n-1)b_{9I}\partial_{\alpha_3}t\partial_{\alpha_4}X^I(\mathcal{F})^{n-2}_{\alpha_5\cdots\alpha_{2n}}\right) = \epsilon^{\alpha_1\alpha_2\alpha_3\cdots\alpha_{2n}\cdots}(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\,. \tag{3.50}$$

In the same way we can see that the last term in (3.49) combine with the first term in (3.49) so that we can replace $C_{\alpha_{2n+1}\cdots\alpha_p}$ with $\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}$. Then (3.49) can be written as

$$aVf'\sum_{n\geq 0}\frac{1}{n!(2!)^n q!}\epsilon^{\alpha_1\cdots\alpha_p x}\left(\partial_K b_{MN}\partial_{\alpha_1}Y^M\partial_{\alpha_2}Y^N(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right). \tag{3.51}$$

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Looking on the form of the expression on the second line in (2.12) it is clear that it can be analysed in the same way as we did above

$$\begin{aligned}
&\epsilon^{\mu_1\cdots\mu_{p+1}}V(T)(\mathcal{F})^n_{\mu_1\cdots\mu_{2n}}\partial_K C_{M_1\cdots M_q}\partial_{\mu_{2n+1}}X^{M_1}\cdots\partial_{\mu_p}X^{M_q}\partial_{\mu_{p+1}}T=\\
&\quad=Vaf'\epsilon^{\alpha_1\cdots\alpha_p x}\Big((\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}\partial_K C_{\alpha_{2n+1}\cdots\alpha_p}+2nb_{9M}\partial_{\alpha_1}t\partial_{\alpha_2}Y^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\partial_K\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}+\\
&\qquad\qquad+q\epsilon^{\alpha_1\cdots\alpha_p x}(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\partial_K C_{x\alpha_{2n+2}\cdots\alpha_p}\partial_{\alpha_{2n+1}}t\Big)
\end{aligned}\tag{3.52}$$

that using the same arguments as were given below (3.49) it can be rewritten in more suggestive form

$$aVf'\epsilon^{\alpha_1\cdots\alpha_p x}(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\partial_K\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\,.\tag{3.53}$$

Now let us consider expression on the third line in (2.12)

$$\begin{aligned}
&-2n\epsilon^{\mu_1\cdots\mu_{p+1}}\partial_{\mu_1}\left[V(T)b_{KM}\partial_{\mu_2}X^M(\mathcal{F})^{n-1}_{\mu_3\cdots\mu_{2n}}C_{\mu_{2n+1}\cdots\mu_p}\partial_{\mu_{p+1}}T\right]=\\
&\quad=-2naf'V\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}Y^M(\mathcal{F})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{\alpha_{2n+1}\cdots\alpha_p}\right]-\\
&\quad-2nVaf'\partial_x[b_{KM}(\tilde{\mathcal{F}})^{n-1}_{\alpha_2\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}]\partial_{\alpha_1}t\partial_{\alpha_2}Y^M-\\
&\quad-4n(n-1)af'V\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}Y^J b_{9N}\partial_{\alpha_3}t\partial_{\alpha_4}Y^N(\tilde{\mathcal{F}})^{n-2}_{\alpha_5\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]-\\
&\quad-2nqaf'V\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}Y^M\partial_{\alpha_3}t(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}C_{x\alpha_{2n+2}\cdots\alpha_p}\partial_{\alpha_{2n+1}}t\right],
\end{aligned}\tag{3.54}$$

where we have used the fact that

$$\epsilon^{\alpha_1\alpha_2\cdots}\partial_{\alpha_1}[Vaf']\partial_{\alpha_2}t=-\epsilon^{\alpha_1\alpha_2\cdots}\partial_x[Vaf']\partial_{\alpha_1}t\partial_{\alpha_2}t=0\,.\tag{3.55}$$

We see that the expression on the first line in (3.54) together with the expression on the third and the fourth line arise from expansions of $\tilde{\mathcal{F}}$ and $\tilde{C}$ in terms of $\mathcal{F},C$ and ∂t. Consequently (3.54) takes the form

$$\begin{aligned}
&-2naf'V\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}Y^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}\right]-\\
&\quad-2nVaf'\partial_x[b_{KM}(\tilde{\mathcal{F}})^{n-1}_{\alpha_2\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+1}\cdots\alpha_p}]\partial_{\alpha_1}t\partial_{\alpha_2}Y^M\,.
\end{aligned}\tag{3.56}$$

Finally, we will analyse the expression on the fourth line in (2.12)

$$\begin{aligned}
&-q\epsilon^{\mu_1\cdots\mu_{p+1}}\partial_{\mu_{2n+1}}\left[V(T)(\mathcal{F})^n_{\mu_1\cdots\mu_{2n}}C_{KM_2\cdots M_q}\partial_{\mu_{2n+2}}X^{M_2}\cdots\partial_{\mu_p}X^{M_q}\partial_{\mu_{p+1}}T\right]=\\
&\qquad=-qaVf'\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_{2n+1}}\left[(\mathcal{F})^n_{\alpha_1\cdots\alpha_{2n}}C_{K\alpha_{2n+2}\cdots\alpha_p}\right]-\\
&\qquad-qaVf'\epsilon^{\alpha_1\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\tilde{C}_{K\alpha_{2n+2}\cdots\alpha_p}\right]\partial_{\alpha_{2n+1}}t-\\
&\qquad-2nqaVf'\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_{2n+1}}\left[b_{9M}\partial_{\alpha_1}t\partial_{\alpha_2}Y^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\cdots\alpha_{2n}}\tilde{C}_{K\alpha_{2n+2}\cdots\alpha_p}\right]-\\
&\qquad-q(q-1)\epsilon^{\alpha_1\alpha_p x}\partial_{\alpha_{2n+1}}\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}C_{Kx\alpha_{2n+3}\cdots\alpha_p}\partial_{\alpha_2}t\right].
\end{aligned}\tag{3.57}$$

Following discussion given below (3.54) we can rewrite (3.57) into the form

$$\begin{aligned}
&-qaVf'\epsilon^{\alpha_1\cdots\alpha_p x}\partial_{\alpha_{2n+1}}\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\tilde{C}_{K\alpha_{2n+2}\cdots\alpha_p}\right]-\\
&\qquad-qaVf'\epsilon^{\alpha_1\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\tilde{C}_{K\alpha_{2n+2}\cdots\alpha_p}\right]\partial_{\alpha_{2n+1}}t\,.
\end{aligned}\tag{3.58}$$

If we look on the expressions in (3.56) and (3.58) we see that there are two terms

$$-2nVaf'\partial_x[b_{KM}(\tilde{\mathcal{F}})^{n-1}_{\alpha_2\cdots\alpha_{2n}}\tilde{C}_{\alpha_{2n+2}\cdots\alpha_p}]\partial_{\alpha_1}t\partial_{\alpha_2}Y^M\,,$$
$$-qaVf'\epsilon^{\alpha_1\alpha_p x}\partial_x\left[(\tilde{\mathcal{F}})^n_{\alpha_1\cdots\alpha_{2n}}\tilde{C}_{K\alpha_{2n+2}\cdots\alpha_p}\right]\partial_{\alpha_{2n+1}}t\,. \tag{3.59}$$

These terms are again important for an interpretation of $t(\xi)$ as an embedding coordinate as was more carefully discussed above. Finally collecting (3.51) , (3.53), (3.56) and (3.58) together we obtain that the current J_K takes the form

$$J_K = af'V\tilde{J}_K\,, \tag{3.60}$$

where $\tilde{J}_K$ is given in (2.19).

As the final point we should determine the form of the current J_9. In fact, since in the analysis performed above there is nothing special about the index K it is clear that the result obtained there can be applied for $K = 9$ as well and we get

$$J_x = af'V\tilde{J}_x\,. \tag{3.61}$$

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